SHRINKAGE EFFECT OF CLOSE COMPONENTS IN SPECTROSCOPIC INSTRUMENTS

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ABSTRACT. Neglecting the Doppler width, the author has calculated the shift of maxima caused by mutual overlapping of intensity patterns of very close spectral lines for a number of spectroscopic instruments. A table, which may be used to correct for shrinkage effect has been given for Fabry Perot etalon. The value of the resolving power of Fabry Perot etalon according to Abbe's criterion has also been refined taking into account the shrinkage effect.

INTRODUCTION

In every spectroscopic instrument, the mutual overlapping of intensity patterns of two very close spectral lines results in their spurious closing together. Very little work has been done on this important aspect, which introduces considerable errors if disregarded, particularly when the instruments are used to the limit of their capacities.

The analysis of the problem becomes extremely difficult when both the Doppler and the instrumental widths are taken into account. Oldenburg (1922) has discussed the case in which there is partial overlapping of the Doppler distribution of two lines. In his treatment he implicitly assumed that the Doppler width is much larger than the instrumental width. Hence this treatment is not applicable to a number of cases in which the natural line width is reduced to be less than the instrumental width, as has been pointed out by Tolansky (1947). A small gap Fabry Perot etalon is generally employed in the examination of a widespread spectrum and the observable line width is mainly due to the instrument and not the line. Similarly with the prism and the grating and even with high resolving power instruments, when modern hyperfine-structure sources like the atomic beam are used, the condition essential to Oldenburg's treatment that the instrumental width be less than Doppler width breaks down.

In this communication the author has calculated the shrinkage effect for a number of instruments taking only the instrumental width into account.

PRISM AND GRATING

The intensity pattern of a spectral line after diffraction by a grating is given by

$$I' = B\frac{\sin^2 N\beta}{\sin^2 \beta} \quad \cdots \quad (1)$$

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where N is the number of lines in the grating and $2\beta = 2\pi\nu e(\sin i - \sin\theta)$ is the the phase difference between the rays diffracted by two adjacent elements of grating, where the symbols have usual meanings.

The maximum intensity say I_0 is given by

$$I_0 = BN^2$$

and hence Eqn (1) may be expressed as

$$\frac{I'}{I_0} = \frac{\sin^2 N\beta}{N^2 \sin^2 \beta} = \frac{\sin^2 x}{x^2} \qquad \ldots \quad (2)$$

where $x = N\beta$ and $\beta \ll 1$.

Eqn (2) also represents the intensity pattern in case of a prism if $x = \pi l\nu \sin\theta$, the symbols having usual meanings.

The quantity Δx is proportional to the angle between two close spectral lines and therefore we shall use Δx instead of $\Delta\theta$ to represent the latter in this investigation. The intensity distribution of another spectral line separated by an angle $\Delta x = a$ is given by

$$\frac{I''}{I_0} = \frac{\sin^2 (x-a)}{(x-a)^2} \qquad \ldots \quad (3)$$

The resultant intensity pattern of two such lines with intensities I_0 and I_0/b (intensity ratio $b:1$) is

$$I/I_0 = (I'+I'')/I_0 = \frac{\sin^2 x}{x^2} + \frac{1}{b}\,\frac{\sin^2 (x-a)}{(x-a)^2} \qquad \ldots \quad (4)$$

For very small values of x we have

$$I/I_0 = \left(1 - \frac{x^2}{3} + \frac{2x^4}{45}\right) + \frac{1}{b}\,\frac{\sin^2 (x-a)}{(x-a)^2} \qquad \ldots \quad (4A)$$

The maximum near $x = 0$, is given by $d\,I/dx = 0$ i.e.

$$\left(-\frac{2x}{3} + \frac{8}{45}x^3\right) + \frac{1}{b}\,F(x) = 0$$

or

$$x = \frac{4}{15}x^3 + \frac{3}{2b}F(x) \qquad \ldots \quad (5)$$

where

$$F(x) = \frac{2\sin^2 (a-x)}{(a-x)^3} - \frac{\sin 2(a-x)}{(a-x)^2}$$

Eqn. (5) may be solved by the method of successive approximations. If x'_n is the solution to n^{th} approximation

$$x'_{n+1} = \frac{4}{15}x'^3_n + \frac{3}{2b}F(x'_n) \qquad \ldots \quad (5a)$$

Similar equation for the shift of maximum of the other line is obtained by putting $1/b$ for b. Thus (5a) becomes—

$$x_{n+1}'' = \frac{4}{15} x_n''^3 + \frac{3b}{2} F(x_n'') \qquad \ldots \quad (6)$$

Putting $x_0' = x_0'' = 0$ we get the first approximation as

$$bx_1' = \frac{x_1''}{b} = \frac{3}{2} F(0) = 3(\sin a/a^2)\{(\sin a/a) - \cos a\} = f(a) \qquad \ldots \quad (7)$$

The higher approximation values can be calculated by the use of equations (5) and (6)

The calculations are particularly valuable when the two lines are close together and when their separation is small compared with the fringe width. In this case first approximation is sufficient and the displacements of two lines from their true positions are given by :

$$\frac{\delta\nu'}{\Delta\nu} = \frac{x'}{a} = \frac{f(a)}{b} \qquad \ldots \quad (8a)$$

$$\frac{\delta\nu''}{\Delta\nu} = \frac{x''}{a} = b\,f(a) \qquad \ldots \quad (8b)$$

where $$f(a) = \frac{3\sin a}{a^3} \left\{\frac{\sin a}{a} - \cos a\right\}$$

$\Delta\nu$ is the separation of the two lines and $\delta\nu'$ and $\delta\nu''$ are the shifts of stronger and weaker components ($b > 1$)

Table I gives the values of $\delta\nu/\Delta\nu$ for values of $a = \pi$, $\pi/1.115$ and $\pi/1.26$ which correspond to the separations required by the Rayleigh, Abbe and Sparrow criteria respectively.

TABLE I

Values of $\delta\nu/\Delta\nu$

b	1/2	1	2	3	4	5
π	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi/1.115$	0.0906	0.0453	0.0227	0.0151	0.0113	0.0091
$\pi/1.26$	—	0.1214	0.0607	0.0405	0.0304	0.0243

For all values of b, $\delta\nu/\Delta\nu = 0$ when $a = \pi$. There will be no maximum, for $b = 1/2$ or less when $a = \pi/1.26$ and for $b = 1/3$ or less when $a = \pi/1.115$, It is interesting to note that $f(a)$ can also assume negative values thereby giving rise to what may be called an expansion effect of close components eg when a is slightly greater than π.

The treatment and results are also applicable to the reflection echelon, which has an intensity pattern similar to the grating.

FABRY PEROT ETALON

The intensity of a spectral line in the order n_0+n, n_0 being an integer and n a small quantity is given by
$I'/I_0 \doteq 1/\{1+F\sin^2\pi(n_0+n)\} = 1/(1+X^2)$ nearly. where $X = \pi F^{\frac{1}{2}}n$, F being the coefficient of fineness and I_0 the intensity of the incident beam.

The intensity pattern of another line of an intensity $1/b$ times that of the first separated by an order Δn is

$I''/I_0 = 1/b\{1+(X-a)^2\}$ where $a = \pi F^{\frac{1}{2}}\Delta n$.

Hence the resultant intensity pattern is given by

$$I/I_0 = (I'+I'')/I_0 = \{1/(1+x^2)\}+[1/b\{1+(x-a)^2\}] \quad \ldots \quad (10)$$

The maxima are given by

$$-(1/2I_0)(dI/dX) = \{X/(1+X^2)^2\}+[(X-a)/b\{1+(X-a)^2\}^2] = 0 \ldots \quad (11)$$

Eqn. (11) is a fifth degree equation and its solution is difficult. Hence we will follow the method of successive approximations. If X'_n is the solution to n-th approximation

$$\frac{X'_{n+1}}{\{1+X'^2_n\}^2}+\frac{X'_{n+1}-a}{b\{1+(X'_n-a)^2\}^2} = 0$$

or

$$X'_{n+1} = a(1+X'^2_n)^2/[(1+X'^2_n)^2+b\{1+(X'_n-a)^2\}^2] \quad \ldots \quad (12)$$

The shift of maximum for the other line x''_n+1 is given by

$$X''_{n+1} = a(1+X''^2_n)^2/[1+X''^2_n)^2+(1/b)\{1+(X''_n-a)^2\}^2] \quad \ldots \quad (13)$$

Taking $X_0' = X_0'' = 0$ we get to the first approximation

$$X' = \frac{a}{\{1+b(1+a^2)^2\}} \text{ and } X'' = \frac{a}{\{1+(1+a^2)^2/b\}}$$

The displacements $\delta\nu'$ and $\delta\nu''$ of the two lines from their true positions are therefore to a first approximation given by

$$\frac{\delta\nu'}{\Delta\nu} = \frac{x'}{a} = \frac{1}{\{1+b(1+a^2)^2\}} \quad (14)$$

$$\frac{\delta\nu''}{\Delta\nu} = \frac{x''}{a} = \frac{1}{\{1+(1+a^2)^2/b\}} \quad \ldots \quad (15)$$

where $\Delta\nu$ is the real wave number difference between the two lines.

Table II gives values of shifts for a range of values of a and b. The values given in the table are such that they differ from the next approximation by less than .005. The table is graphically illustrated in figure 1. The corrections for shift of maxima may be applied with the help of the table, and the graphs by

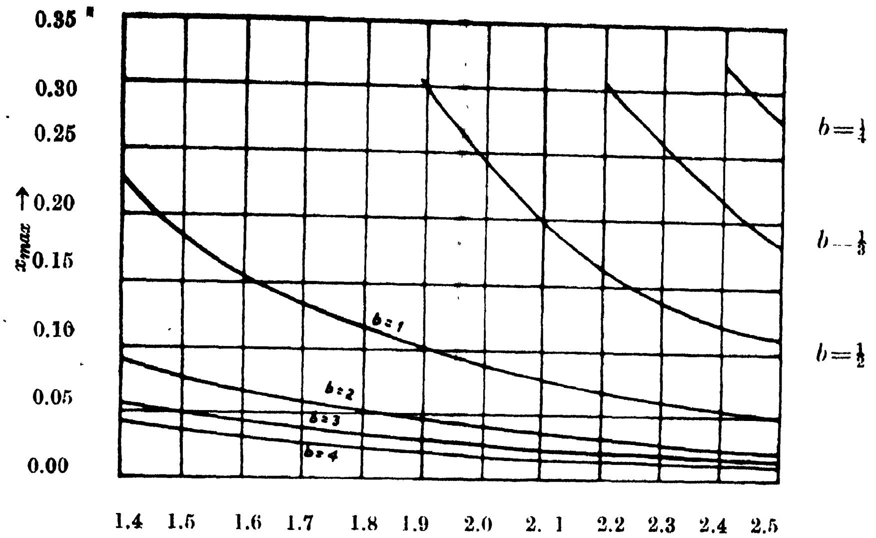

Fig. 1

the method of successive approximations, i.e. noting the corrections for the measured values of a, adding them to a and again determining the corrections.

TABLE II

Values of shift of maxima (x) in F. P. etalon.

a \ b	1/4	1/3	1/2	1	2	3	4	5
1.4				0.2286	0.0900	0.0572	0.0420	0.0312
1.5				0.1815	0.0786	0.0504	0.0370	0.0279
1.6				0.1550	0.0680	0.0442	0.0328	0.0249
1.7				0.1342	0.0560	0.0391	0.0294	0.0222
1.8				0.1170	0.0531	0.0347	0.0258	0.0198
1.9			0.3000	0.1020	0.0471	0.0308	0.0229	0.0177
2.0			0.2398	0.0895	0.0418	0.0263	0.0198	0.0156
2.1			0.2030	0.0778	0.0373	0.0236	0.0178	0.0143
2.2		0.3060	0.1610	0.0693	0.0317	0.0213	0.0160	0.0128
2.3		0.2620	0.1399	0.0619	0.0287	0.0192	0.0144	0.0116
2.4	0.3194	0.2150	0.1232	0.0555	0.0259	0.0174	0.0131	0.0105
2.5	0.2752	0.1830	0.1080	0.0500	0.0236	0.0158	0.0118	0.0095